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Analysis of the Use of Finite Element Modeling (FEM) to Simplify the Design Equation for Web Tapered-I Beams

Nakul Misra[1], Dr. Gaurav Shukla[2]

[1]Research Scholar, Department of Civil Engineering, School of Engineerging & Technology, Maharishi University of Information Technology, Lucknow, Uttar Pradesh (esotericnakul@gmail.com)
[2]Assistant Professor, Department of Civil Engineering, School of Engineerging & Technology, Maharishi University of Information Technology, Lucknow, Uttar Pradesh (gaur.knit@gmail.com)

Abstract: The web-tapered I-columns exhibit the ability to withstand flexural buckling and lateral torsional buckling at specific locations, while the capacity in other areas of the member is comparatively reduced. It is necessary to focus on the non-prismatic members to determine the buckling capacity, and standard procedures should be established in accordance with Indian Code IS 808:2021. This research investigates simulated finite element models encompassing 60 web tapered column sections with taper ratios (h2/h1) ranging from 1.0 to 3.0, utilizing FEA software ANSYS. This research presents a comprehensive Eigen-value buckling analysis, resulting in a novel design equation for the calculation of the buckling load of web tapered I columns. This equation is capable of predicting the buckling stress for any taper ratio of web tapered I columns of varying lengths.

Keywords: Finite Element Modelling (FEM), Tapered-I Beams, Stress Analysis, ANSYS Structural Stability, Eigen Value Buckling

Introduction: This study conducted by Prawel. S. P. identifies that the construction industry and steel manufacturers globally are currently prioritizing material savings, optimization, and economic design, particularly with the emergence of three-dimensional printing technologies. The tapered steel members have been in use since the 1960s, as reported, and the design provisions were conservative during a time when computational solutions were not available[1].

The Joint Task Committee initiated the formation of the Column Research Committee and the Welding Research Council in 1966. The tapered structural elements underwent destructive testing in earlier research phases to determine bending and buckling strength. The boundary conditions were adjusted to ensure that failure occurred within the elastic limit.

This study was conducted by Butler. D.J. et al. found that these testing programs represented a continuation of welding research. The research history regarding the fabrication of Tapered I sections in the 1960s typically involved two distinct methods: plates produced through the shearing process and plates cut using oxygen cutting techniques. Previous research indicates that the buckling effect exhibited greater intensity with an increase in the angle of taper. Non-uniform tapered sections exhibit greater efficiency compared to prismatic sections and are suitable for applications where major axis bending fluctuates along the length of the beam[2].

Narrow rectangular structural elements exhibit susceptibility in flexural and torsional stiffness, while web tapered I columns and beams demonstrate the opposite, with minimal impact on minor flexural rigidity and sensitivity confined to warping rigidity. Closed-form solutions exist for non-prismatic tapered I-sections. Diverse numerical methods have been referenced for determining the elastic buckling loads of tapered I sections, accompanied with solutions[3-7].

Industrial halls, warehouses, and exposition centers often make use of tapered sections due to the structural efficiency and attractive appearance they provide. Under the h2/h1 member design taper ratio of 4, a greater amount of the current structure is covered. Worldwide, the steel construction industry has been striving for more efficient, less expensive, and quicker production of industrial sheds, trusses, bridges, multi-story buildings, factory structures, housing units, and prefabricated modules. Because of its structural efficiency, which resulted in material savings and low weight, tapered members have a wider application in the steel industry.

Aesthetics and reasonable fabrication costs, made possible by the rise of 3D printing, would be the result. Web tapered I sections, in contrast to the remainder of the member, have an exceptional capacity to resist flexural buckling and lateral torsional buckling at a specific place, which is either not necessary or is significantly lower[8].

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capacity to resist flexural buckling and lateral torsional buckling at a specific place, which is either not necessary or is significantly lower. There is no mention of web tapered non prismatic members in the IS 808: 2021 Indian Steel Code. Liliana explains that the EC3 has established rules for checking the structural integrity of prismatic-membered frames and elements. It is necessary to further understand and codify the behavior of non-prismatic members with tapers and castellation. The buckling strength of web tapered I-beams and columns has been the subject of numerous research proposals and derivations. However, every theory requires computations, which are based on time-consuming non-linear buckling analysis and repetitions. Very few studies have used the Rayleigh-Ritz method to determine the critical buckling load of tapered-I columns[9].

We found discrepancies of up to 8% when we compared it to the Linear Buckling Analysis Model. A new method for conducting buckling analyses of tapered column members was disclosed by Rahai, the researcher. A tapering index was suggested as an addition to the critical buckling load estimation formula in the research. The mode forms of tapered columns were determined by linearly combining different modes of prismatic cross sections. A better distribution of the moment of inertia along the length was also found to improve the critical buckling stresses[10]. Ioannis analyzed the critical lateral-torsional buckling moments of web-tapered I beams. The stability analysis of tapered I beams was conducted. The investigations indicated that the tapered beams experienced maximum moment at a singular location. The scientist notes that numerous studies have focused on the behavior of tapered beams within the elastic limit. The research findings revealed theoretical insights that may be integrated into codal provisions. Slope deflection was employed to predict the buckling capacity of tapered bar elements subjected to flexural and axial forces. Research was conducted on flexural torsional buckling under transverse and axial loads[11].

The research findings indicate that stability analysis is essential for the design of web tapered I beams or columns. The study indicated that the taper ratio (h2/h1) significantly affects the determination of the critical load in web tapered I beams, especially in cantilever configurations. The h2 represents the depth of the “I” column at the point of fixity. The h1 represents the depth of the I section at the top of the column, where h2 is greater than h1. This paper presents the formulation of a design equation aimed at predicting the critical buckling load of web tapered I sections utilized as columns. The tapering ratios were adjusted to 1, 1.5, 2, 2.5, and 3.0. This study focuses on the Eigen value buckling analysis of web tapered I column sections utilizing ANSYS. The influence of taper ratio on critical buckling capacity, specifically Euler's buckling load, has been extensively examined. A total of 60 web tapered column sections with 5 taper ratios (h2/h1) ranging from 1.0 to 3.0 were simulated in ANSYS using shell elements (SHELL 63), and their critical buckling stress was determined. The buckling behavior was restricted to three specific lengths: 6h, 7h, and 8h, which were not addressed in previous studies. The boundary conditions were assumed to exhibit fixity at one end and freedom at the opposite end. The most susceptible boundary conditions were utilized. Research indicates that the taper ratio significantly influences the critical buckling stress of web tapered columns. The results are presented graphically, and the research proposes a new expression for determining the critical buckling stress in web tapered column sections.

(A) Web Tapered I-Beams:

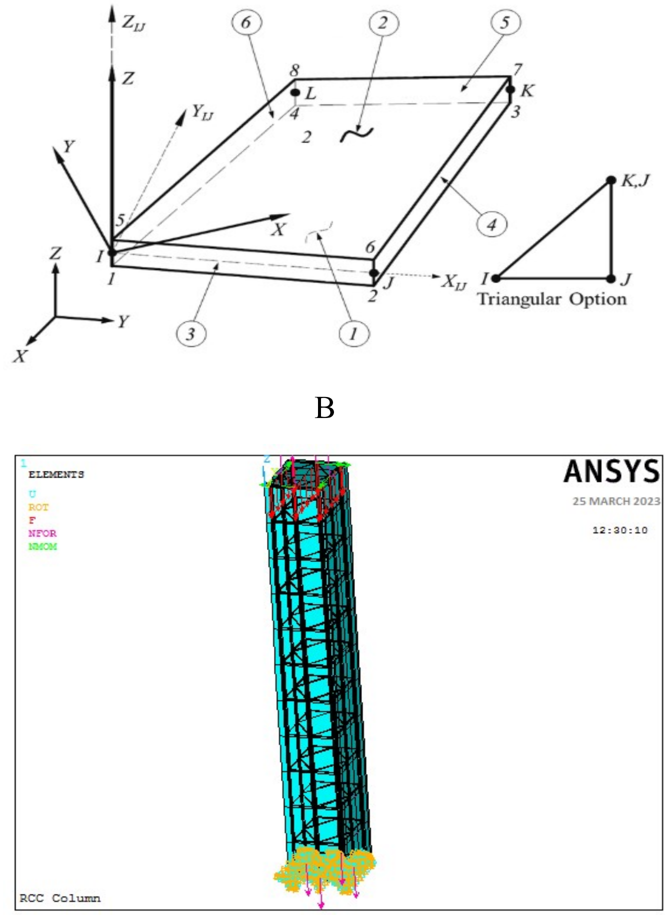
This research selects the (Indian Standard Medium Weight Beams) ISMB 200, ISMB 300, ISMB 350, and ISMB 400. For each "I" section, five taper ratios were utilized for simulations with lengths of 6h, 7h, and 8h. The characteristics of the sections were referenced from SP 6 – 1 (1964) (Handbook for Structural Engineers). The selected sections were prismatic standard sections from the handbook, while the attributes were determined based on the relevant slenderness ratios. The medium weight "I" beam sections were selected for this pilot investigation, and rigorous research was conducted[12-13].

(B) Web-tapped I Beams simulated in ANSYS:

SHELL63 pieces were used to represent the web tapering columns. This SHELL 63 is capable of both membrane and bending. Normal loads as well as in-plane loads are allowed. At each node, the element has six degrees of freedom: rotations about the nodal x, y, and z-axes and translations in the nodal x, y, and z directions. Included are substantial deflection capabilities and stress stiffening.

For large deflection (finite rotation) investigations, a constant tangent stiffness matrix option is offered. The tapered columns were modeled using varying taper ratios (h2/h1) but the same flange thickness (tf) and web thickness (tw). The shell element featured four nodes, each of which was given the same thickness, as seen in figure 1 (a). Two thicknesses, denoting tf and tw, were used to define the mapped meshing employing SHELL 63 elements. The SHELL elements had been adopted with two sets of real constants: one for the flange thickness and another for the web thickness. Every tapering column was simulated using the skinning process.

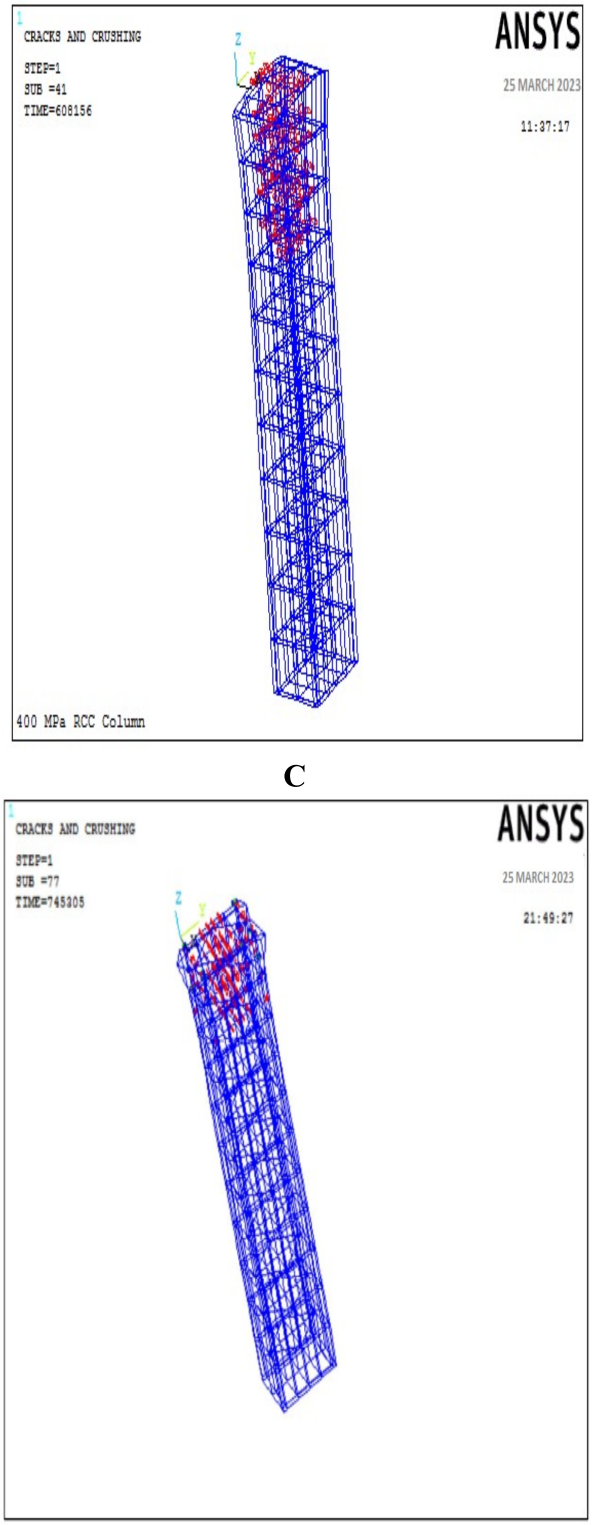
The material model was given steel's characteristics. The simulated web tapered I section, shown in figure 1 (b), shows the application of loading at the top and the limitations at the bottom (fixed support). In the figure above, the web taper was visible. After crucial points and lines were created, the simulation was made possible. The 3D simulation resulted from the lines' skinning. The simulated model that depicted the tapered I column was created by meshing the web and the flanges, then merging the common nodes to preserve continuity. The buckling strength of the columns was then determined by doing the Eigen value buckling analysis.



B

A

Figure 1 (A and B) displays the SHELL 63 element and Loads and Constraints



C

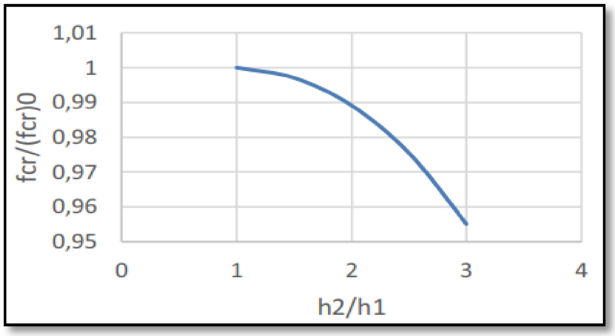
D

Figures 2 (C and D) show the ISMB 400 strain plot and stress plot, respectively

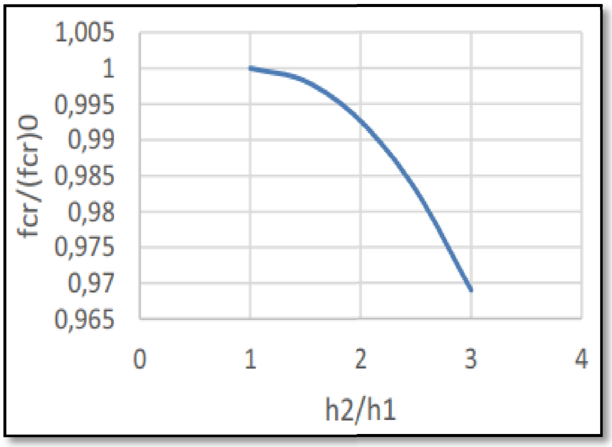
The strain curve of ISMB 400 is displayed in figure 2 (C). The best value was 0.539 e-4, while the bottom value was 0.741 e-7. The fixity condition was observed at the bottom, where the highest stress of -0.0155 MPa was shown in figure 2 (D). According to the ANSYS17.2 Eigenvalue buckling study, the load factor for ISMB 400 is 579.04. The plots displayed in both figures are examples of the analysis that was conducted using FEA software. The same simulation method was used for other areas as well.

Results and Discussions:

For ISMB200, ISMB300, ISMB350, and ISMB400, the Eigen value analysis was performed. With slenderness ratios of 6h, 7h, and 8h, and five taper ratios of 1.0, 1.5, 2.0, 2.5, and 3.0, each tapered I column was unique. So, over sixty specimens had been modelled for the five parts. You may find a summary of the reviewed research in the following tables and pie chart. A graph showing the critical load ratio (fcr/fco) as a function of the taper ratio in each column for various L/h ratios is shown.

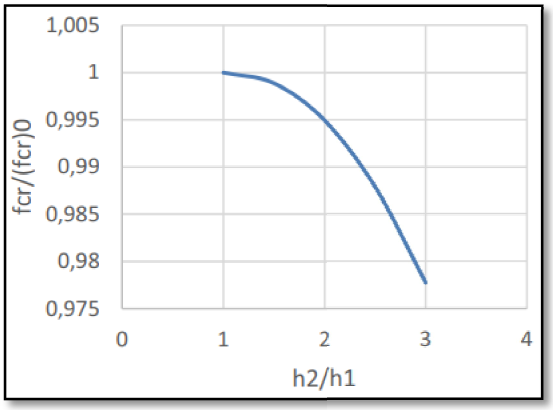


E

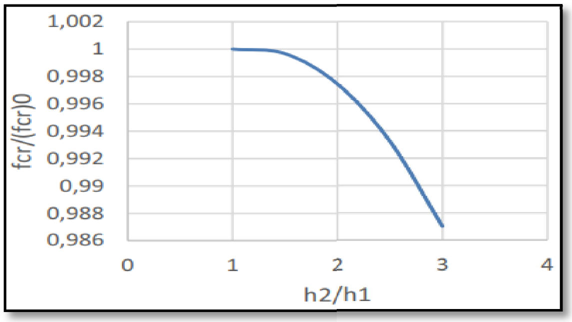


F

Figure 5 (E and F) shows the relationship between taper ratio and buckling stress for ISMB 200 at lengths of 6h and 7h



G



H

Figure 6 (G and H) shows the relationship between taper ratio and buckling stress for ISMB 200 (l=8h) and 300 (l=6h) at lengths

The values 5 (E and F) and 6 (G and H) shown the relationship between buckling stress and taper ratio for ISMB 200. The fcr denotes the critical buckling stress of the tapered section as determined by Eigenvalue Buckling analysis on the tapered section. The fcr0 represents the buckling stress for the prismatic column constructed from ISMB 200.

The graphical representation may indicate a decrease in both buckling stress and buckling load as the length of the tapered column rises. A decline of 4% was noted in ISMB 200 for a duration of 6 hours, and a decrease of 3.1% was recorded for 2.2% at L = 8 hours. The 4% decline is consistent with the results observed by Ioannis G. Ratoyiannis et al. for lengths exceeding 6h.

Table 1 shows the result of the curve fit that was generated from the graphical displays. (See Table 1) for the equations that describe the relationship between the taper ratio and the buckling stress ratio.

ISM	**6h**	**7h**	**8h**
B 200	$\frac{fcr}{fco} = 1.007\left(\frac{h_2}{h_1}\right).^{-0.038}$	$\frac{fcr}{fco} = 1.005\left(\frac{h_2}{h_1}\right).^{-0.268}$	$\frac{fcr}{fco} = 1.004\left(\frac{h_2}{h_1}\right).^{-0.02}$
ISM B 300	$\frac{fcr}{fco} = 1.023\left(\frac{h_2}{h_1}\right).^{-0.011}$	$\frac{fcr}{fco} = 1.056\left(\frac{h_2}{h_1}\right).^{-0.029}$	$\frac{fcr}{fco} = 1.004\left(\frac{h_2}{h_1}\right).^{-0.206}$
ISM B 350	$\frac{fcr}{fco} = 1.0031\left(\frac{h_2}{h_1}\right).^{-0.053}$	$\frac{fcr}{fco} = 1.0051\left(\frac{h_2}{h_1}\right).^{-0.026}$	$\frac{fcr}{fco} = 1.0037\left(\frac{h_2}{h_1}\right).^{-0.018}$
ISM B 400	$\frac{fcr}{fco} = 1.004\left(\frac{h_2}{h_1}\right).^{-0.019}$	$\frac{fcr}{fco} = 1.004\left(\frac{h_2}{h_1}\right).^{-0.017}$	$\frac{fcr}{fco} = 1.003\left(\frac{h_2}{h_1}\right).^{-0.016}$

Table 1 shows the correlation between buckling stress and taper ratio

Table 1 shows the correlation between buckling stress and taper ratio for all sections ranging from ISMB200 to ISMB400. The trend exhibits minimal variation across all instances. The disparities among all the equations may be insignificant.

The variation is presumed to be parabolic, and the convergence point is nearly same for all tapered columns whose attributes were derived from SP. From the aforementioned finding, the below equation is constructed, which serves as a simplified model for forecasting the buckling stress of tapered "I" beams.

The buckling stress can be utilized to determine the critical buckling load as illustrated in equations A and B.

$$fcr = 1.0023\frac{\pi^2 E}{(1000\lambda)^2}\left(\frac{h_2}{h_1}\right)^{-0.011} \qquad \text{Equations: A}$$

$$P_{cr} = f_{rc}\ X\ A_1 \qquad \text{Equations: B}$$

Conclusions:

Finite Element Modeling (FEM) has demonstrated efficacy in streamlining the design equations of web tapered-I beams. FEM simulations precisely capture intricate stress distributions and deformation patterns, obviating the necessity for unduly conservative or simplified assumptions inherent in classic analytical approaches. A detailed study of analysis was conducted utilizing FEA with SHELL 63 elements in ANSYS. The element was suitable and practical for the buckling analysis. The integrated direct matrix abstraction program utilized the "Block Lanczos" algorithm for linear buckling analysis within the FEA software. The pre-stressing effect was activated upon reaching the buckling load utilizing a single mode. Approximately 60 specimens of web tapered I sections have been simulated with SHELL 63 elements. The slenderness ratio exerts a negligible effect on the prediction of buckling stress. The height of the web has minimal influence on the buckling stress, exhibiting a linear and negligible change. The taper ratio greatly influences buckling stress; as the taper ratio increases, the buckling stress ratio decreases, as indicated by EC3-1-1. The study developed a straightforward formula for calculating the critical Euler load for tapered I columns, derived from the buckling stress obtained from eigen-value buckling analysis. The formula is suitable for assessing critical stress in non-prismatic web tapering sections. The formula's accuracy was assessed according to previous study predictions articulated by Ioannis G. Raftoyiannis. The values are precise when juxtaposed with the aforementioned conservative formulas. The finite element modeling can determine the critical buckling load. This research has streamlined the application and formulation for assessing buckling stress in web tapered columns.

Conflicts of Interest:

The authors declare that they have no conflicts of interest.

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